2009–05–19

Lecturer: Bengt E W Nilsson

Recall: The standard model without supersymmetry from D6-branes in type IIA string theory, with compactification on T^6. Compare Heterotic $E_8 \times E_8$ on Calabi–Yau $\Rightarrow N = 1$, $D = 4$ standard model, which is roughly the MSSM.

Last time: we wanted gauge groups and fermion spectrum to follow from some D6-brane configuration (including 3 families).

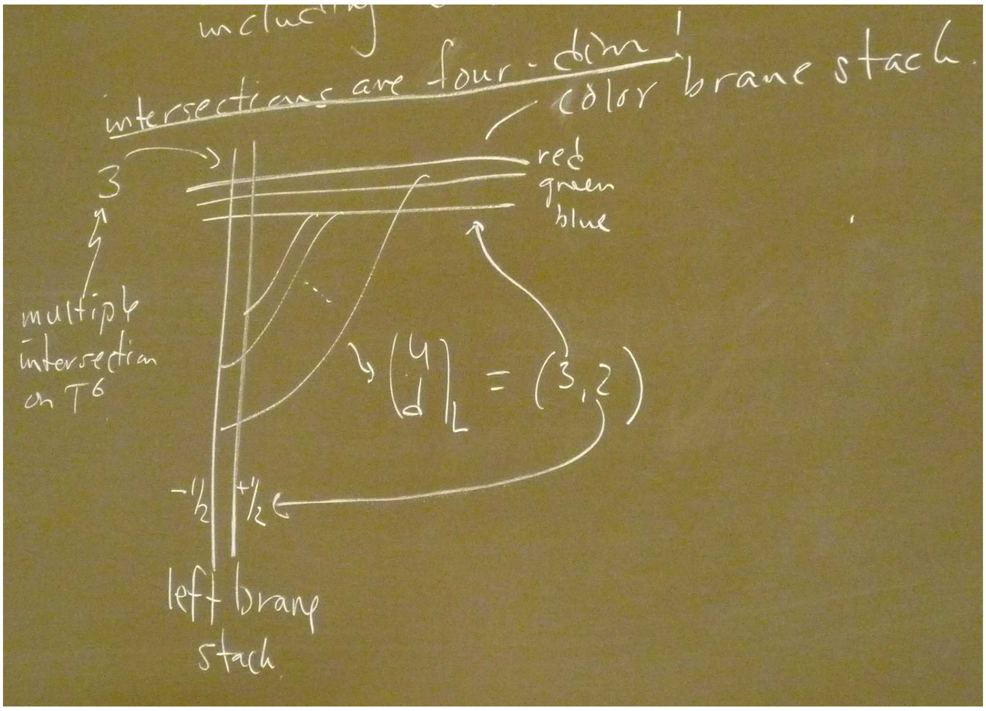

Figure 1.

This picture will only give the exact Standard Model if orientifolds are also used.

21.5: String theory models and particle physics

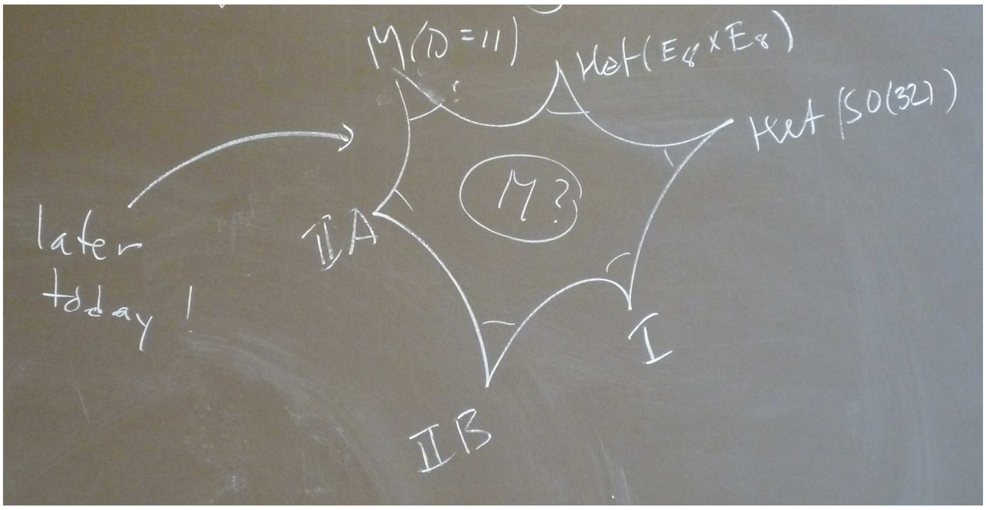

Figure 2. You can't draw these in any order you want; they are related in this fashion. We will study one of the connections today.

21.6: Moduli stabilisation

String theory on $S^1(R)$ contains a dynamical scalar field $R(x)$ in spacetime whose vacuum expectation value $\langle R(x)\rangle = R = \text{const.}$

How is the value $\langle R(x)\rangle$ determined?

1) In the simple cases $\text{IIA}|_{T^6}$ the radii are not determined $\Rightarrow$ Moduli space of theories parametrised by six radii. ($\mathbb{R}^6$). There is no potential $V(R)$ yet.

2) Can we introduce a $V(R)$ in a natural way? Yes!

Example: $D=6 \rightarrow D=4+2$-dimensional compact manifold:

$$-\,\mathrm{d}s^2 = \underbrace{g_{\mu\nu}\,\mathrm{d}x^\mu \mathrm{d}x^\nu}_{D=4} + R^2(x)\,\bar{g}_{ab}(y)\,\mathrm{d}y^a \mathrm{d}y^b$$

$R(x)$: The scale of the internal manifold depends on $x^\mu \in$ spacetime.

Reducing Einstein gravity in $D=6$ to $D=4$ using this metric gives an ordinary Lagrangian, containing a potential

$$V(R) = -\,a_g \frac{\chi}{R^4}, \quad \begin{cases} a_g > 0 \\ \text{Euler number } \chi = 2-2\,g \end{cases}$$

(After Weyl rescaling of the metric!)

$$\Rightarrow \begin{cases} g=0 \;\Rightarrow\; V(R)<0 \;\Rightarrow\; R\rightarrow 0 \;\; (S^2 \text{ vanishes! Not stable!}) \\ g=1 \;\Rightarrow\; V=0 \qquad\qquad \text{moduli space of stable solutions} \\ g>1 \;\Rightarrow\; V(R)>0 \;\Rightarrow\; R\rightarrow\infty. D=4\rightarrow D=6 \text{ decompactification (unstable)} \end{cases}$$

3) Can we stabilise the radius R? I.e., can we find ways to produce more terms in $V(R)$? Yes. Introduce fluxes. With a magnetic flux of $\Phi = 2\pi n, n\in\mathbb{Z}$.

$$\Rightarrow \quad B = \frac{n}{R^2}$$

$\Rightarrow$ Energy density $\sim R^2 B^2 \sim n^2/R^2$. But then the Weyl rescaling gives also a $1/R^4$ factor.

$$V_{\text{new}}(R) = a_{\text{f}}\,\frac{n^2}{R^6}$$

So

$$V(R) = -\,a_g\,\frac{\chi}{R^4} + a_{\text{f}}\,\frac{n^2}{R^6}$$

$\Rightarrow S^2$ radius is now stabilised.

Note: for $\chi=0$ we must add also other objects like orientifolds and D-branes.

The Witten argument from 1995

IIA string theory, $D=10$:

NS–NS: $g_{\mu\nu}, B_{\mu\nu}, \phi$

R–R: $A_\mu, A_{\mu\nu\rho}$

NS–R, R–NS: $2\times\psi_\mu$ (spin $\frac{3}{2}$), $2\times\lambda$.

We would like to understand these in $D=11$, concentrating on NS–NS and R–R. If you look at the two fields g_{MN} and A_{MNP} you can produce the $D=10$ fields.

$$g_{MN}=\begin{pmatrix} g_{\mu\nu} & A_\mu \\ A_\mu & \phi \end{pmatrix}$$

$$A_{MNP}=(A_{\mu\nu\rho}, A_{\mu\nu 11})$$

We identify $A_{\mu\nu 11}=B_{\mu\nu}$.

Can we write down a Lagrangian in $D=11$? Yes, and it is unique! The Lagrangian is the low energy approximation to M-theory.

$$L^{(D=11)}=\frac{e}{2\kappa}R-\frac{1}{24!}(F_{MNPQ})^2-\frac{\mathrm{i}}{2}e\,\bar{\psi}_M\Gamma^{MNP}\mathrm{D}_N\psi_P+$$

where we have introduced $F_{MNPQ}=\partial_M A_{NPQ}+3$ terms, $e=\sqrt{-g}$. $g_{\mu\nu}=e^\alpha_\mu e^\beta_\nu\eta_{\alpha\beta}\Rightarrow\sqrt{-g}=\det e^\alpha_\mu=e$. From the ordinary γ matrices you can define $\gamma^{\mu\nu}\equiv\frac{1}{2}(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu)$.

$$+\frac{\sqrt{2}\,\kappa}{(12)^4}e\,\varepsilon^{M_1\ldots,M_{11}}F_{M_1M_2M_3M_4}F_{M_5\ldots M_8}A_{M_9M_{10}M_{11}}+$$

$$+\frac{3\kappa}{4\sqrt{2}(12)^2}e\big(\bar{\psi}_M\Gamma^{MNPGRS}\psi_N+12\bar{\psi}^P\Gamma^{\mathrm{QR}}\psi^S\big)\Big(F_{PQRS}+\tilde{F}_{PQRS}\Big).$$

Now we want to compactify on a circle to 10 dimensions.

$$-\,\mathrm{d}s^2=g_{\mu\nu}(x)\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu+\underbrace{\mathrm{e}^{2\gamma(x)}}_{\substack{R^2(x)\\ \text{before}}}\big(\mathrm{d}x^{11}-A_\mu\,\mathrm{d}x^\mu\big)^2$$

where x^{11} is the circle coordinate. A straight forward, but tedious calculation, will produce

$$L^{D=10(\mathrm{comp})}_{\substack{\text{bosonic}\\ \text{sector}}}=\frac{1}{4\kappa^2}e\,\mathrm{e}^\gamma R-\frac{1}{4}e\,\mathrm{e}^{3\gamma}(F_{\mu\nu})^2-\frac{1}{12}e\,\mathrm{e}^{-\gamma}(F_{\mu\nu\rho})^2-\frac{1}{48}e\,\mathrm{e}^\gamma(F_{\mu\nu\rho\sigma})^2$$

Comments:

- there is no $\partial\gamma\partial\gamma$ kinetic term.
- $e_{D=11}\rightarrow e_{D=10}\,\mathrm{e}^\gamma$
- e^γ from $e_{D=11}$, $g^{\mu\rho}_{D=11}\,g^{\nu\sigma}_{D=11}\,\partial_\mu g^{D=11}_{\nu 11}\,\partial_\rho g^{D=11}_{\sigma 11}\,g^{11,11}_{D=11}$

$$g^{\mu\rho}_{D=11}\rightarrow g^{\mu\rho},\quad g^{\nu\sigma}_{D=11}\rightarrow g^{\nu\sigma},\quad g^{D=11}_{\nu,11}A_\nu\mathrm{e}^{2\gamma},\quad g^{11,11}_{D=11}\rightarrow\mathrm{e}^{-2\gamma}$$

$$\Rightarrow\mathrm{e}^{3\gamma}$$

Looking at the Einstein term it seems like the $D=10$ Newton's constant depends on the field $\gamma(x)$. To eliminate this dependence (and compare to physics) we *Weyl rescale* it away:

Take the metric $g_{\mu\nu}$ and e^γ, the two fields, and Weyl rescale: $\tilde{g}_{\mu\nu}=\mathrm{e}^{-2s\gamma(x)}g_{\mu\nu}$ for a parameter s. The result is, with a,b being parameters in L

$$\mathrm{e}^{-2a\gamma}\,\tilde{e}\Big(\tilde{R}-b(\partial_\mu\gamma)^2\Big)=\mathrm{e}^{-(2a+s(D-2))\gamma}\,e\,\Big(R+\big(s^2(D-1)(D-2)-4\,a\,s(D-1)b\big)(\partial_\mu\gamma)^2\Big)$$

Weyl rescale Witten's answer in $D=10$ using $s=\frac{1}{8}$. This will produce the $D=10$ canonical

$$L^{D=10(\mathrm{can})}=\frac{1}{4\kappa^2}\,e\,R-\frac{9}{32}e(\partial_\mu\gamma)^2-\frac{1}{4}\,e\,\mathrm{e}^{9\gamma/4}(F_{\mu\nu})^2-\frac{1}{12}\,e\,\mathrm{e}^{-3\gamma/2}(F_{\mu\nu\rho})^2-\frac{1}{48}e\,\mathrm{e}^{3\gamma/4}(F_{\mu\nu\rho\sigma})^2.$$

In fact, using instead $S=\frac{3}{8}$ we get a Lagrangian that is connected to the type IIA string:

$$L^{D=10(\mathrm{string})}=\frac{1}{4}e\,\mathrm{e}^{-3\gamma}\Big(R+9(\partial_\mu\gamma)^2\Big)-\frac{1}{4}\,e\,(F_{\mu\nu})^2-\frac{1}{12}\,e\,\mathrm{e}^{-3\gamma}(F_{\mu\nu\rho})^2-\frac{1}{48}e\,(R_{\mu\nu\rho\sigma})^2$$

So set $\mathrm{e}^{-3\gamma}=\mathrm{e}^{-2\phi}$ where ϕ is the dilaton of the type IIA string. Then $L^{D=10(\mathrm{string})}$ becomes exactly the Lagrangian computed in type IIA string theory.

But now $\langle e^{\gamma}\rangle\equiv R$, $\langle\mathrm{e}^{\phi}\rangle\equiv g_s$

$$\Rightarrow g_s=R^{3/2}$$

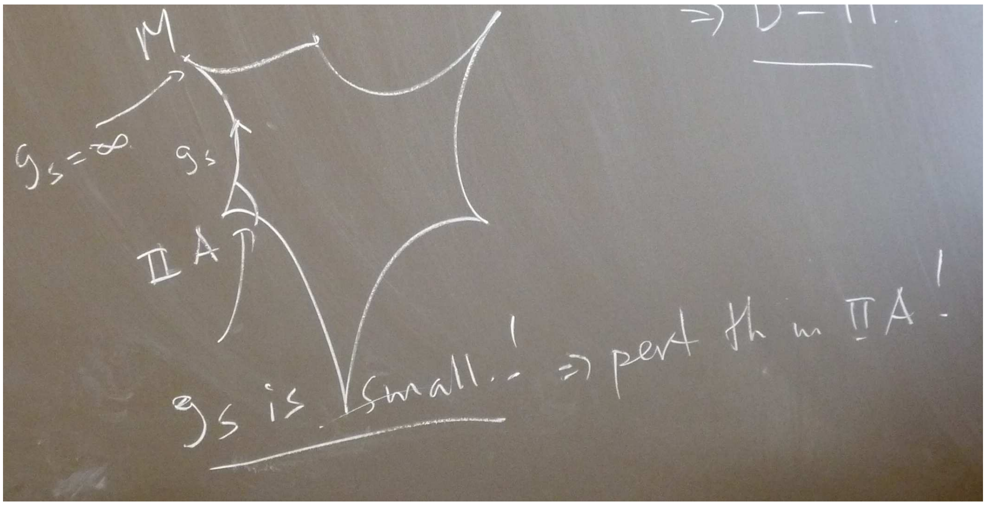

Figure 3.

$g_s\to\infty\Rightarrow R\to\infty$, which takes us back to $D=11$.

Maldacena: AdS/CFT.