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Lecturer: Per Salomonson

Standard model of elementary particles

Background history.

Before the standard model existed, four types of interactions were recognised.

1. Gravity.

2. Electromagnetism.

3. Weak interaction.

4. Strong interaction.

Strong interaction and weak interaction discerned: Weak violates symmetries, like P, C, T and others. Responsible for decays, for example $n \rightarrow p + e + \bar{\nu}$.

The standard model handles everything but gravity. The interactions are described in the standard model as gauge interactions. Matter was divided into (A) hadronic matter: protons, neutrons, nuclei, ... and (B) leptonic: e, μ, ν_e, n_μ, ... In addition, we have radiation energy, γ, gravitons. Hadronic matter is strongly interacting, leptonic is not. In the Standard Model hadrons are described by quarks, spin $\frac{1}{2}$ fields. Leptons are also spin $\frac{1}{2}$ fields. In addition we have Higgs fields, spin 0.

Gauge principle for constructing theory, Standard Model of elementary particles defined by field content and Lagrangian. $\mathcal{L}$ constructed using gauge principle.

1. Define matter fields.

2. Write $\mathcal{L}_0 =$ the Lagrangian density of the free matter fields.

3. Choose continuous symmetry of $\mathcal{L}_0$, that is to be made into the gauge symmetry.

4. Gauge this symmetry.

5. The Lagrangian density of the Standard Model is the most general renormalizable Lagrangian of the given fields, with the chosen gauge group, and perhaps with some additional global symmetry.

Examples:

1) Electrodynamics.

$$\mathcal{L}_0 = \bar{\psi}\left(\mathrm{i}\,\partial\!\!\!/ - m\right)\psi \quad \psi = \text{electron field}$$

Continuous symmetry group: $G = \mathrm{U}(1)$: $\psi \rightarrow \mathrm{e}^{-\mathrm{i}e\Lambda}\,\psi$. Infinitesimally $\delta\psi = -\,\mathrm{i}\,e\,\Lambda\psi$.

$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\,D\!\!\!\!/ - m\right)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \quad \mathrm{D}_\mu = \partial_\mu + \mathrm{i}\,e\,A_\mu, \quad \delta A_\mu = \partial_\mu\Lambda$$

$$\Rightarrow \mathrm{D}_\mu\psi = \mathrm{e}^{-\mathrm{i}e\Lambda}\mathrm{D}_\mu\psi$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad \delta\,F_{\mu\nu} = 0$$

Note that $\mathcal{L}$ contains all renormalizable gauge invariant terms.

$$\left[\,\bar{\psi}\,\sigma^{\mu\nu}\psi F_{\mu\nu}\right] = M^{2\times\frac{3}{2}+2} = M^5$$

Note: Trouble with $\partial_\mu \psi$ and $\psi \to \mathrm{e}^{-\mathrm{i}e\Lambda(x)}\psi$ is that $\partial\psi = [\psi(x+\Delta x) - \psi(x)]/\Delta x$ compares the value of ψ at different points in space, and we rotate by a different amount Λ in different points.

2) Strong interactions.

We use quark fields:

$$\mathcal{L}_0 = \sum_{a=1}^{3} \bar{q}^{\,a}\left(\mathrm{i}\partial\!\!\!/ - m_q\right) q_a \equiv \bar{q}\left(\mathrm{i}\partial\!\!\!/ - m\right) q$$

q_a is the quark field. It is a Dirac spinor. a is colour index.

Gauge symmetry $G_S = \mathrm{SU}(3)_c$.

$$q_a \to \sum_{b=1}^{3} U_a{}^b q_b \equiv U q$$

$U =$ unitary 3×3 matrix, $U^\dagger U = \mathbf{1}$.

4. $U \to U(x)$

Gauge transformations: $q \to U(x)\, q$

$$\partial_\mu q \to \partial_\mu (Uq) = (\partial_\mu U) q + U \partial_\mu q = U(\partial_\mu q + \underbrace{U^\dagger(\partial_\mu U)}_{\text{unwanted}}) q$$

Cure: replace derivative by covariant derivative, $\mathrm{D}_\mu q = (\partial_\mu + A_\mu) q$

$$(\partial_\mu + A_\mu) q \to U(\partial_\mu + A_\mu) q$$

requires $A_\mu q \to U(A_\mu - U^\dagger \partial_\mu U) q$, i.e. $A_\mu \to U A_\mu U^\dagger + (\partial_\mu U) U^\dagger$.

$$U U^\dagger = 1 \quad \Rightarrow \left(\partial_\mu U^\dagger\right) + U\left(\partial_\mu U^\dagger\right) = 0$$

Then $\bar{q}\left(\mathrm{i}\, D\!\!\!/ - m\right) q \equiv \bar{q}\,(\mathrm{i}\,\gamma^\mu(\partial_\mu + A_\mu) - m) q$ is OK as term in $\mathcal{L}$ (gauge invariant).

Are there other terms?

Note: $q \to U q$. $D_\mu q \to U D_\mu\, q$. This means that $\mathrm{D}_\mu \to U D_\mu U^\dagger = U\,(\partial_\mu + A_\mu) U^\dagger$.

NB: here ∂_μ acts on everything to the right. Also

$$[\mathrm{D}_\mu, \mathrm{D}_\nu] = \mathrm{D}_\mu \mathrm{D}_\nu - \mathrm{D}_\nu \mathrm{D}_\mu \to U[\mathrm{D}_\mu, \mathrm{D}_\nu] U^\dagger$$

Note: here free derivative operators cancelled. This implies that $\mathrm{tr}[\mathrm{D}_\mu,\, \mathrm{D}_\nu]^n$ is gauge invariant, and e.g. $\mathrm{tr}([\mathrm{D}_\mu,\, \mathrm{D}_\nu][\mathrm{D}^\mu,\, \mathrm{D}^\nu])$ is gauge invariant and Lorentz invariant; thus acceptable in $\mathcal{L}$. Thus

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}\left(\mathrm{i}\, D\!\!\!/ - m\right) q + \frac{1}{8g^2}\,\mathrm{tr}(G_{\mu\nu} G^{\mu\nu})$$

(QCD: the theory of strong interactions is called quantum chromodynamics), where

$$G_{\mu\nu} = [\mathrm{D}_\mu, \mathrm{D}_\nu] = (\partial_\mu + A_\mu)(\partial_\mu + A_\nu) - (\partial_\nu + A_\mu)(\partial_\mu + A_\mu) =$$

(Here ∂_μ is free derivative operator, acts on everything to the right.)

$$= \partial_\mu A_\nu - \partial_\nu A_\mu + A_\mu A_\nu - A_\nu A_\mu$$

(Here ∂ acts only on A.) and $\mathrm{D}_\mu = \partial_\mu + A_\mu$.

$$A_\mu \rightarrow U A_\mu U^\dagger + \partial_\mu U U^\dagger$$

A_μ shall be thought of as an infinitesimal SU(3) object, i.e. Lie algebra object.

$$U = \mathrm{e}^{\mathrm{i}\varepsilon H} = 1 + \mathrm{i}\,\varepsilon\, H + \mathcal{O}(\varepsilon^2), \quad U^\dagger U = \mathbf{1} \quad \Leftrightarrow \quad H^\dagger = H$$

SU(3) matrix can be expressed using Hermitian matrix.

$$A_\mu \rightarrow (1 + \mathrm{i}\varepsilon H) A_\mu (1 - \mathrm{i}\varepsilon H) + (\partial_\mu (1 + \mathrm{i}\varepsilon H))(1 - \mathrm{i}\varepsilon H) = A_\mu + \mathrm{i}\varepsilon\,(H A_\mu - A_\mu H + \partial_\mu H)$$

$$\delta A_\mu = \mathrm{i}\,(H A_\mu - A_\mu H + \partial_\mu)$$

It is consistent to choose A_μ antihermitian. Common notation, expand A_μ in a basis:

$$A_\mu = \sum_{a=1}^{8} t_a\, A_{a\mu}$$

where $\{t_a\} =$ basis of antihermitian 3×3 matrices, chosen traceless for SU(3) (the S property of **S**U(3)).

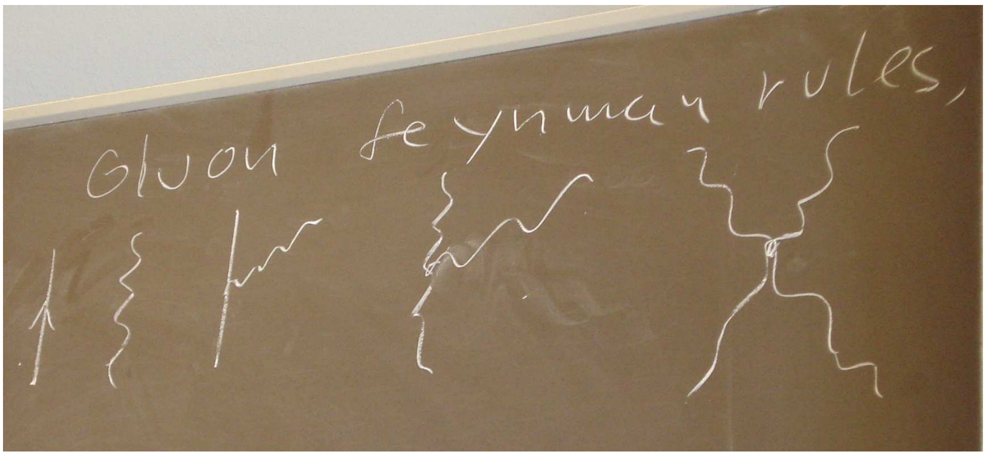

Figure 1. Gluon Feynman rules.

Photons are electrically neutral, but gluons have colour.

Asymptotic freedom.

First: scale dependence of couplings.

First QED;

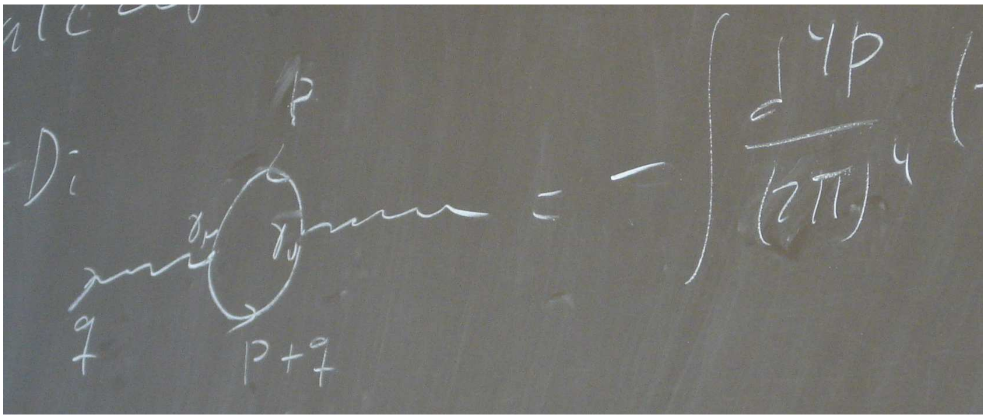

Figure 2.

$$\langle \mathrm{Fig2}\rangle = \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,(-\mathrm{i}\,e)^2\,\mathrm{tr}\!\left(\gamma_\mu \frac{\mathrm{i}}{\not{p}+q-m}\,\gamma_\nu \frac{\mathrm{i}}{\not{p}-m}\right) = \mathrm{i}\,\Pi(m,q,\Lambda)_{\mu\nu} = \mathrm{i}\,\Pi_{\mu\nu}$$

Property $q^\mu \Pi_{\mu\nu} = 0$ by current conservation.

$$\Rightarrow \Pi_{\mu\nu} = \left(\eta_{\mu\nu} q^2 - q_\mu q_\nu\right) f(m,q,\Lambda)$$

$$[f] = M^0$$

Calculation shows:

$$f = \frac{e^2}{2\pi^2}\ln\frac{m^2+q^2}{\Lambda^2} + \cdots = \frac{e^2}{2\pi^2}\left(\ln\!\left(\frac{m^2+q^2}{\mu^2}\right) - \ln\!\left(\frac{\Lambda^2}{\mu^2}\right)\right)$$

$$\langle 0|T A_\mu(x)\,A_\nu(0)\,\mathrm{e}^{-\mathrm{i}\int \mathrm{d}^4 x'\mathcal{H}_I}|0\rangle = \langle \mathrm{Fig3}\rangle \qquad \frac{\mathrm{i}\eta_{\mu\nu}}{q^2} + \frac{-\mathrm{i}}{q^2}\,\mathrm{i}\,\Pi_{\mu\nu}\,\frac{-\mathrm{i}}{q^2} + \cdots$$

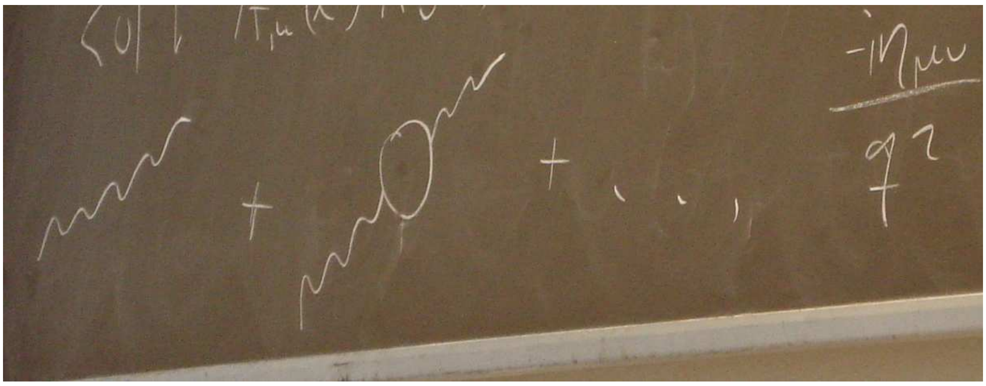

Figure 3.

Renormalize $A_{\mu\nu}$:

$$\mathcal{L}_0 = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \rightarrow -\frac{1}{4}Z\,F_{\mu\nu}\,F^{\mu\nu}$$

$\Rightarrow$ propagator $1/q^2 \rightarrow Z/q^2$. Z: $1 + \frac{e}{12\pi} - \ln\frac{\Lambda^2}{\mu^2}$.

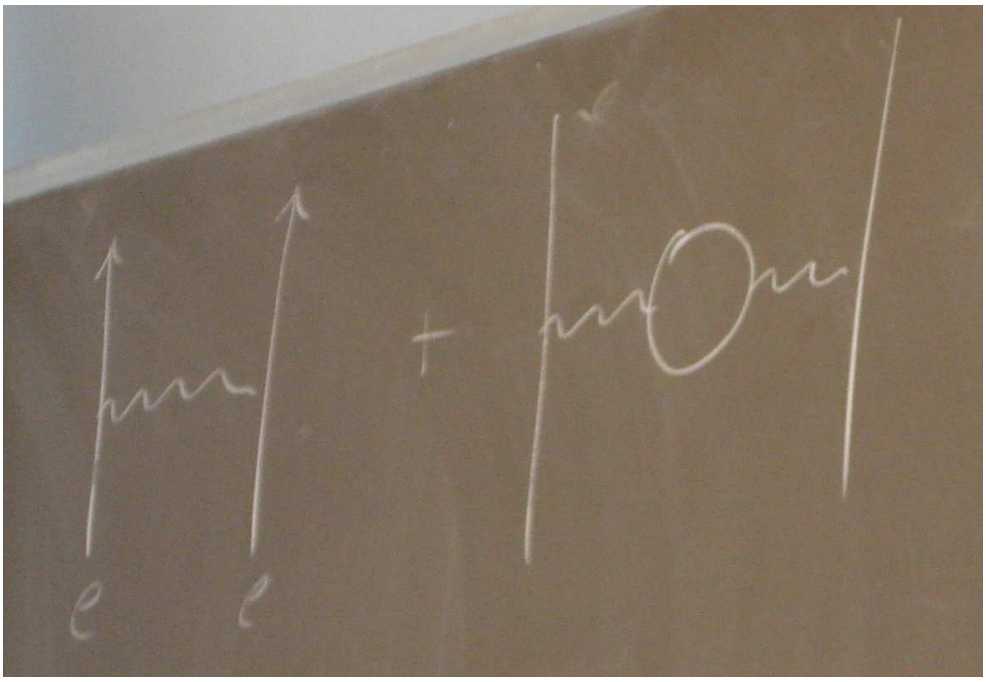

Figure 4.

$$\langle\text{Fig4}\rangle \quad \bar{u}\gamma_\mu u\,\bar{u}\gamma_\nu u\,(-\mathrm{i}e)^2(\ldots)\eta^{\mu\nu}\left(\frac{\mathrm{i}}{q^2}+\frac{\mathrm{i}}{q^2}\,\mathrm{i}\,q^2\,f(m,q,\Lambda)\,\frac{\mathrm{i}}{q^2}\right)\ldots$$

Now cutoff dependence can be absorbed into renormalisation of e. Scale dependent electric charge. Result:

$$e^2(q)\approx\frac{e_0^2}{1-\frac{e_0^2}{12\pi^2}\ln\left(\frac{m^2+q^2}{m^2}\right)}$$

where e_0 is the electron charge found in textbooks.

Effective charge increases with energy scale = increases when objects become close.

Fourier transform coulomb potential

$$\delta^3(r)=\int\frac{\mathrm{d}^3q}{(2\pi)^3}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}=-\frac{1}{4\pi}\,\nabla^2\,\frac{1}{r}$$

$$\frac{e^2}{4\pi r}=\int\frac{\mathrm{d}^3q}{(2\pi)^3}\,\frac{e^2}{q^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}}$$

For strong interactions the effective interaction change weakens at high momenta.