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$m\,\bar{e}_R\,e_R =$

$$e_R = \frac{1}{2}(1+\gamma_5)\,\psi$$

$$\bar{e}_R = e_R^{\dagger}\gamma^0 = \frac{1}{2}\,\psi^{\dagger}(1+\gamma_5)\gamma^0 = \frac{1}{2}\,\underbrace{\psi^{\dagger}\gamma^0}_{=\bar{\psi}}\,(1-\gamma_5)$$

$$\left\{\gamma^0,\gamma^5\right\} = 0;\quad \gamma^5 = \mathrm{i}\,\gamma^0\gamma^1\gamma^2\gamma^3$$

$m\,\bar{e}_R\,e_R = m\frac{1}{2^2}\,\bar{\psi}\,(1-\gamma_5)(1+\gamma_5)\psi$, and $(1-\gamma_5)(1+\gamma_5) = 0$. $m\,\bar{e}_R\,e_R = 0$.

$$m\,\bar{\psi}\,\psi = m(\bar{e}_R\,e_L + \bar{e}_L\,e_R)$$

In order to make a Dirac mass you need a Dirac spinor. This is possible in ordinary Dirac theory, but it is not allowed in the Standard Model, because of SU(2)

$$L_L = \begin{pmatrix} \nu_L \\ e_R \end{pmatrix}$$

Let us remind ourselves the content of the Standard Model.

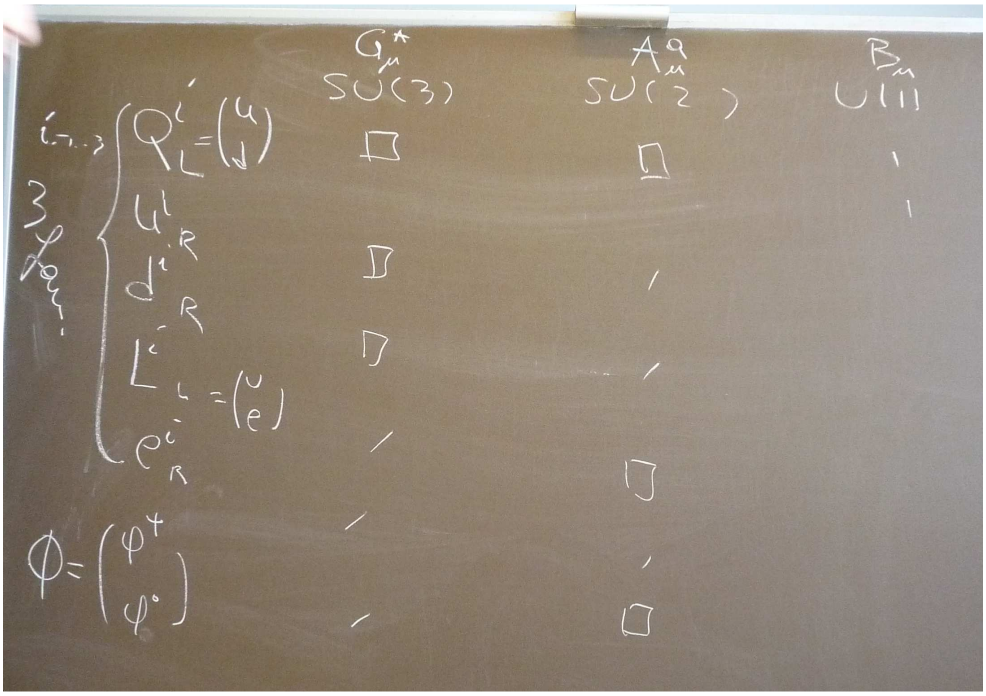

Figure 1.

$$L_L^a\,\Phi_a\,e_R$$

$\delta\psi = \mathrm{i}\,g\,\tau^A\psi$ where $\left[\tau^A,\tau^B\right] = \mathrm{i}\,f^{ABC}\,\tau^C$. f^{ABC} fixes the algebra. Define $\tilde{\tau}^A = -\,(\tau^A)^T$ (transpose, not dagger — $\tau^{A\dagger} = \tau^A$).

$$\left[\tilde{\tau}^A,\tilde{\tau}^B\right] = \mathrm{i}\,f^{ABC}\,\tilde{\tau}^C$$

This is called a conjugate representation.

Take another field, so that $\delta\psi_a = \mathrm{i}\, g\, \alpha^A(x)\, \tau^A{}_a{}^b\ \psi_b$ and $\delta\chi^a = \mathrm{i}\, g\, \alpha^A(x)\, \tilde{\tau}^A{}_b{}^a\, \chi^b$. The fields with index up transform in the one representation, the fields with index down transform in the other representation.

$$\delta\psi\chi = \delta(\psi_a\chi^a) = 0$$

$$\begin{array}{cccl} \bar{L}_L^a & \Phi_a & e_R & \\ +\frac{1}{2} & +\frac{1}{2} & -1 & = 0 \end{array}$$

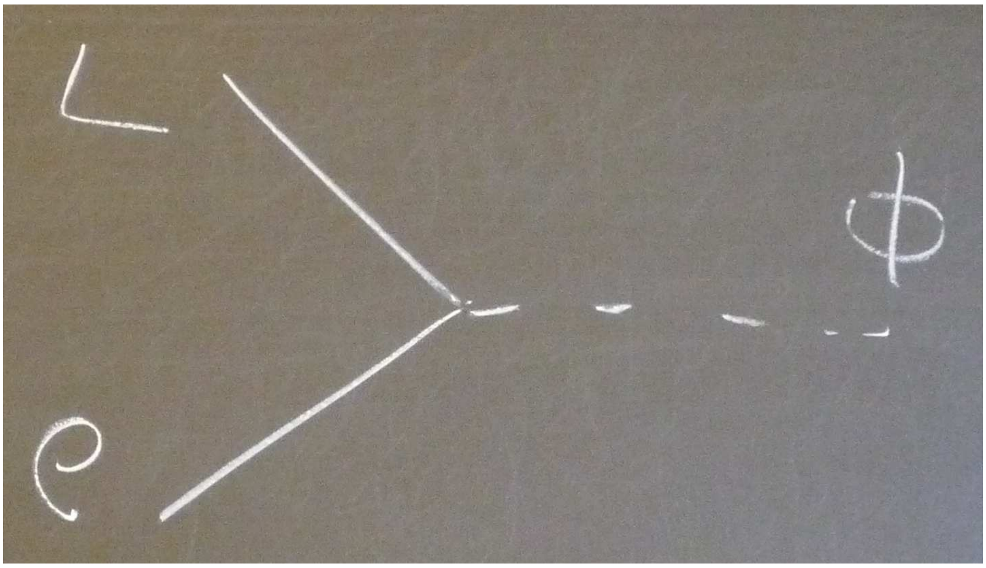

Figure 2. L, e, Φ

$$F^2 + \bar{\psi}\, \not{D} + (\mathrm{D}\phi)^2 + \bar{\psi}\phi\psi + \phi^2 + \phi^4$$

$$\text{“}\bar{\psi}\phi\psi\text{”} = -\lambda_{(d)}^{ij}\, \bar{Q}_L^{\mathrm{ai}} \phi_a\, d_R^j - \lambda_{(u)}^{ij}\ \varepsilon_{ab}\, \bar{Q}_L^{ia}\, \phi^{\dagger b}\, u_R{}^j$$

Yukawa coupling λ: 3×3 matrix with no symmetry whatsoever. ε_{ab} is $\varepsilon_{11} = \varepsilon_{22} = 0, \varepsilon_{12} = -\varepsilon_{21} = 1$.

$$\phi^b \rightarrow g^b{}_c\, \phi^c, \quad Q^a \rightarrow g^a{}_b\, Q^d$$

$$\varepsilon_{ab}\, \phi^b\, Q^a \rightarrow \underbrace{\varepsilon_{ab}\, g^b{}_c\ g^a{}_d}_{= \det(g) \cdot \varepsilon_{cd}}\, \phi^c\, Q^d$$

The determinant in SU(2) is one.

$$\text{“}\bar{\psi}\phi\psi\text{”} = -\lambda_{(d)}^{ij}\, \bar{Q}_L^{\mathrm{ai}} \phi_a\, d_R^j - \lambda_{(u)}^{ij}\ \varepsilon_{ab}\, \bar{Q}_L^{ia}\, \phi^{\dagger b}\, u_R{}^j - \lambda_c^{ij} \bar{L}_L^{ia} \phi_a e_R^j \overset{?}{-} \lambda_\nu^{ij}\, \varepsilon_{ab}\, \bar{L}_L^{ia} \phi^{\dagger b} \nu_R^j$$

$$\text{“}\phi^2 + \phi^4\text{”} = -V(\phi)$$

$\phi \rightarrow g_{2\times 2}\, \phi, \phi^\dagger \rightarrow \gamma^\dagger g^\dagger$. $\phi^\dagger \phi \rightarrow \phi^\dagger\ g^\dagger g\, \phi = \phi^\dagger \phi$.

$$\text{“}\phi^2 + \phi^4\text{”} = -V(\phi) = \mu^2\, \phi^\dagger \phi - \lambda\big(\phi^\dagger \phi\big)^2$$

There are several minima. Choose $\phi_0 = \frac{1}{\sqrt{2}} \left(\begin{smallmatrix} 0 \\ v \end{smallmatrix} \right), v$ real

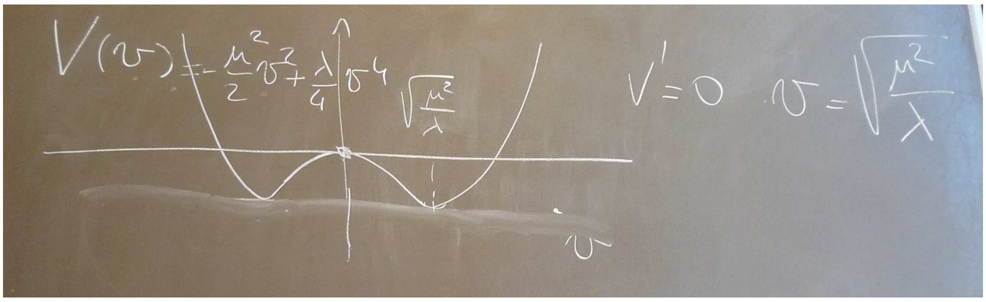

Figure 3.

$v' = 0$

$$246\,\mathrm{GeV} = v = \sqrt{\frac{\mu^2}{\lambda}}$$

$$\mathrm{D}_\mu \phi_0 = \frac{1}{\sqrt{2}} \left(\partial_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} - \mathrm{i}\, g\, A^a_\mu \tau^a \begin{pmatrix} 0 \\ v \end{pmatrix} - \mathrm{i}\, g' \frac{1}{2} B_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} \right) =$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \begin{pmatrix} -\mathrm{i} g\, A^3_\mu - \mathrm{i} g'\, B_\mu & -\mathrm{i}\, g \big(A^1_\mu - \mathrm{i} A^2_\mu \big) \\ -\mathrm{i}\, g \,\big(A^1_\mu + \mathrm{i} A^2_\mu \big) & +\mathrm{i}\, g\, A^3_\mu + \mathrm{i}\, g'\, B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

$$|\mathrm{D}_\mu \phi_0|^2 = \frac{1}{8} \begin{pmatrix} 0 & v \end{pmatrix} \begin{pmatrix} g\, A^3_\mu + g'\, G_\mu & g \big(A^1_\mu - \mathrm{i}\, A^2_\mu \big) \\ g \big(A'_\mu + \mathrm{i}\, A^2_\mu \big) & -g\, A^3_\mu + g'\, B_\mu \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} =$$

$$= \frac{v^2}{8} \Big(g^2 \big(A^1_\mu + \mathrm{i}\, A^2_\mu \big) \big(A^1_\mu - \mathrm{i}\, A^2_\mu \big) + \big(-g\, A^3_\mu + g'\, B_\mu \big)^2 \Big)$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}} \big(A^1_\mu \pm \mathrm{i}\, A^2_\mu \big)$$

$$Z^0 = \frac{1}{\sqrt{g^2 + g'^2}} \big(g\, A^3_\mu - g'\, B_\mu \big)$$

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \big(g'\, A^3_\mu + g\, B_\mu \big)$$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} A^3 \\ B \end{pmatrix}$$

$$\mathcal{L}_{\substack{\text{gauge} \\ \text{mass}}} = \underbrace{\frac{v^2 g^2}{4}}_{m_W^2} W^+_\mu W^{\mu -} + \underbrace{\frac{v^2}{8} \left(g^2 + g'^2 \right)}_{\frac{1}{2} m_Z^2} Z_\mu Z^\mu$$

$$\mathrm{D}_\mu\, e_R = (\partial_\mu - \mathrm{i}\, g' \,(-1) B_\mu)\, e_R = \left(\partial_\mu + \mathrm{i}\, \frac{g'}{\sqrt{g^2 + g'^2}} (g'\, Z_\mu + g\, A_\mu) \right) e_R$$

$$|e| = \frac{g\, g'}{\sqrt{g^2 + g'^2}} = g \sin\theta_W = g' \cos\theta_W$$

$$\mathrm{D}_\mu Q_L = \left(\partial_\mu - \text{gluons} - \frac{\mathrm{i}\, g}{\sqrt{2}} \left(W^+ \tau^+ + W^- \tau^- \right) - \mathrm{i}\, \frac{g}{\cos\theta_W}\, Z_\mu \big(\tau^3 - \sin^2\theta_W \mathcal{Q} \big) - \mathrm{i}\, e\, A_\mu \mathcal{Q} \right) Q_L$$

$$\mathcal{Q} = \tau^3 + Y$$

$e\mathcal{Q}$ is the electric charge.

$$\text{“}\bar{\psi}\phi_0\psi\text{”} = -\lambda_d^{ij}\,\bar{Q}_L^{ia}\phi_a\,d_R + \cdots$$

$$= \lambda_{(d)}^{ij}\Big(\not{\bar{u}}{}_L^i, \bar{d}_L^i\Big)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} d_R^i = -\lambda_{(d)}^{ij}\cdot\frac{1}{\sqrt{2}}\,v\cdot\bar{d}_L^i\,d_R^j$$

Invariant under SU(3) and $\mathrm{U}(1)_{\mathrm{EM}} \subset \mathrm{SU}(2)\times\mathrm{U}(1)$.

$$\mathcal{Q}d_L = -\frac{1}{2} + \frac{1}{6}$$

$$\mathcal{Q}d_R = 0 - \frac{1}{3}$$

$$\mathcal{L}_{\substack{\text{fermi}\\ \text{masses}}} = -\frac{v}{\sqrt{2}}\Big(\lambda_{(d)}^{ij}\,\bar{d}_L^i\,d_R^i\Big) + \lambda_{(u)}^{ij}\,\bar{u}_L^i\,u_R^i + \lambda_{(e)}^{ij}e_L^2\,e_R^j + ?$$

Not the physical ones (yet). You have to diagonalise the quadratic part of the Lagrangian.